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# Percolation and fracture

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A statistical model of fracture, based on percolation theory, is presented which allows the quantitative evaluation of clustering of cracks in solids. Unlike models of branching processes which are more in accord with Griffith fracture, the concept of percolation lattices (finite and infinite) is used following from a physical model of multiple fracture.

Some experimental results on acoustic emission, dilatancy and geophysical precursors of earthquakes can be correlated with the percolation fracture model. The model does not depend on the mechanism of crack formation, critical parameters being the number of elementary events (cracks), the dimensionality of the process and the coordination number for a network of cracks and, in finite systems, their specific size.

Fracture prediction is possible from the number of elementary acts, cluster statistics and other characteristic parameters of the model. Possible applications of the percolation model for earthquake prediction are considered.

## 1. Introduction

In limiting-state theories, the strength of a solid is generally characterised by the ultimate (breaking) load which is required to overcome the cohesion forces of immobile atoms and, besides a deterministic approach (i.e. Griffith, Irvin, Orovan, Eshelby and Dagdeil, see Liebowitz, 1972), statistical theories have been developed according to which strength is characterised by a distribution function (Weibull, 1939; Bolotin, 1961; Yokobori, 1966; Rikitake, 1976; Stiller et al., 1979). However, as experiments have shown, strength is substantially dependent on duration of loading, which is explained by the decisive role of thermal fluctuations in the destruction process. Taking thermal fluctuation statistics into consideration, one can pass from an engineering parameter, i.e. 'strength', to a fundamental conception of durability or 'lifetime' of solids under constant loading which forms the basis for kinetic theory of strength (see Zhurkov and Nazrullaev, 1953; Regel et al., 1974); but 'durability' is a limiting value too, and it does not describe fracture development.

At the same time, in connection with earthquake prediction problems, models are required that can reflect the development of this process. In earlier theories describing the evolution of fracture (Kachanov, 1974) an abstract function, 'medium damage rate', was preset, which was not connected with main elementary phenomena of the process such as microfractures, their concentration and orientation, degree of clustering, etc. For this reason, over the last few years, models have been intensively advanced which consider characteristics of microcracks. Thus, to describe fracture as a developing process theory of random processes is used, e.g. theory of branching (Markov) processes (Bolotin, 1971; Otsuka, 1972; Vere-Jones, 1977; Petrov, 1979; Pathwardhan et al., 1980), as well as percolation theories (Shante and Kirkpatrick, 1971; Shklovski and Efros, 1979). These methods of description are in many ways similar. Thus, for example, Gayda and Ottavi (1974) have shown

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that the theory of branching processes and calculations of the percolation model with one initial point lead to identical conditions for cluster interpenetration into infinity: coordination number of sites in a sphere of radius $r_c$ should be not less than 2.7 (or 4.4 in a circle of the same radius for a two-dimensional model). It is evident that the model of branching processes (as well as percolation trees) is in closer conformity to the problem of the probabilistic description of the propagation of one crack through the whole system (as evidenced, for example, in Vere-Jones, 1977) than with the picture of disperse fracture.

It is the purpose of this paper to develop a percolation model of multiple fracture where destruction is the consequence not of the propagation of a single crack, but is a result of the merging of a set of elementary cracks which have no selected centre of propagation and which appear to be randomly distributed in space.

Probably, Otsuka (1972) and Vere-Jones (1976) were the first to note the possibility of the application of percolation theory to the description of fracture, the latter again being treated as growth of a single crack. It should be noted, however, that such an approach has not been developed much further due to difficulties of strict analytical treatment (Vere-Jones, 1976). Undoubtedly, for the time being percolation theory is faced with difficulties, though in some cases it has been possible to obtain rigorous solutions (Chirkov, 1976). This paper will try to show that, despite this, a percolation model of fracture has many positive features, which make it a rather useful tool for modelling and prognosing the process of destruction and its statistical interpretation.

## 2. Basic definitions of percolation theory

Percolation theory (Hammersley and Broadbent, 1957; Shante and Kirkpatrick, 1971) describes an interaction of a flow with a disordered medium (which may be imagined as a system of sites) depending on the concentration of inhomogeneities (fraction of performed sites) $p$. The increase of concentration of such sites results in cluster formation. Numerical experiments on the percolation theory conducted by the author, among others, have shown that the number of finite clusters $N_{cl}$ (unit defects included) increases with $p$ at first linearly up to $p = p_a$ when aggregates begin to appear (twins, triplets, etc.), then, naturally, more slowly; at $p = p_{am}$ the number of clusters passes through a maximum and begins to lessen with $p$ growth. This means that, at $p = p_{am}$, the process of new cluster formation is dominated by the process of merging of defects into clusters. It has been found (Chelidze and Kolesnikov, 1982) that $p_a \simeq 0.08$ and $p_{am} \simeq 0.26$ for two-dimensional square lattices.

Such behaviour of function $N_{cl}(p)$ can be accounted for by the fact that, in the percolation theory, statistics with 'memory' are used, i.e. when a performed site causes an irreversible change of the situation, in contrast to conventional statistics, where the probability of event is constant and is independent of the number of performances. At a critical value $p = p_c$, i.e. at the so-called 'percolation threshold' which is the key parameter of the theory, finite clusters merge and an infinite cluster (IC) is formed that passes through the whole system.

The percolation process can be described by several characteristic functions, e.g. percolation probability, $W(p)$

$$W(p) = 0 \quad \text{at} \quad p < p_c;$$
$$W(p) = (p - p_c)^{\beta} \text{ at } \quad p > p_c \qquad (1)$$

mean number of sites in finite clusters, $S(p)$

$$S(p) \simeq |p - p_c|^{-\gamma} \qquad (2)$$

correlation length $R(p)$

$$R(p) \simeq |p - p_c|^{-\nu}/p_c \qquad (3)$$

As the behaviour of functions $N_{cl}$, $S$ and $R$ depends, in some way, on approaching the percolation threshold, they can be considered as precursors of formation of an infinite cluster (Fig. 1). In eqs. 1–4 exponents $\beta$, $\gamma$, $\nu$ are the so-called 'critical indices' defined by the dimensionality of the system, $D$ such that: for $2D$, $\gamma_2 = 2.38$, $\beta_2 = 0.14$, $\nu_2 = 1.33$, and for $3D$, $\gamma_3 = 1.69$, $\beta_3 = 0.35$, $\nu_3 = 0.8$–$0.9$. Critical indices $\beta$, $\gamma$ and $\nu$ are related by: $\nu D = \gamma + 2\beta$.

Percolation threshold $p_c$ is also defined by the

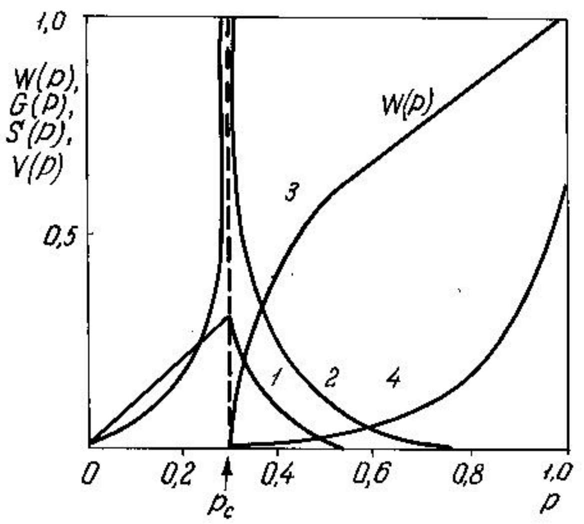


Fig. 1. Characteristic percolation functions versus concentration of bonds $p$: (1) volume fraction of finite clusters $V(p)$; (2) mean number of sites in finite clusters $S(p)$ (the curve of correlation radius $L(p)$ is similar); (3) volume fraction of infinite clusters or percolation probability $W(p)$; (4) conductivity $G(p)$.

system dimensionality and by the site distribution pattern. If $z$ is a coordination number of connecting bonds for the given site, then for $z$ varying from 3 to 12 the following expression is valid: $p_c \simeq D/z(D-1)$. For convenience, the dimensionality of the system will be denoted below by superscripts $2D$ or $3D$.

The percolation theory, as well as the theory of critical phenomena, describes changes of various properties of matter, i.e. electrical conductivity (hopping), ferromagnetism, fluid flow in porous media, by the same equations and specificity of the process and the substance is given mostly by the values of critical indices. Such general character formulae attest to the fact that relatively large-scale topological peculiarities of the media, and not microscopic (atomic) ones, determine the form of characteristic functions (1)–(3), by means of which such different physical characteristics of matter as conductivity, penetrability or susceptibility are approximated.

## 3. Physical background for a percolation model of destruction

According to qualitative physical models, a process of multiple rupture in solids develops as follows: at first, single isolated microcracks appear and then with the increase of load or time of loading the density of cracks per unit volume increases, they begin to merge and at last at a 'critical density of cracks' (Scholz, 1968; Brady, 1974, 1975, 1976a, 1976b; Tamuzh and Kuksenko, 1978) the main fracture is formed, i.e. disintegration of the body takes place. The similarity of these physical phenomena with the percolation theory, of 'critical crack density' with percolation threshold, main fracture with an infinite cluster is readily apparent (see also Vere-Jones, 1976).

A simple physical model corresponding to the 'classical' isotropic percolation theory can be built as follows. Using a site lattice with characteristic size $L$ and lattice constant l approximate a body. At first, all sites are intact, corresponding to an ideal defectless body. Then, at random, lattice sites 'spoil' one by one, as is postulated by a kinetic theory of multiple fracture (Zhurkov and Nazrullaev, 1953; Regel et al., 1974) according to which microcracks form as a result of very large thermal fluctuations.

Suppose that the volume fraction (density) of elementary cracks corresponds to the fraction of the performed sites $p$ in the percolation theory; by this is meant the effective volume fraction that includes a crack stress field presented in a first approximation by a sphere with radius $r$. Assume that all elementary cracks are the same and their 'radius of action' is of the order of the lattice constant l and that cracks developed in neighbouring sites of the lattice form a cluster, i.e. a large-size fracture. Clearly, such growth of clusters corresponds to precritical sizes of cracks as all clusters, both large and small, grow in a similar random manner. This situation can be realized in highly-inhomogeneous bodies and in highly plastic materials. If the concentration of cracks $p$ increases continuously, then, at a certain moment, overlapping fields of clusters of cracks form an infinite cluster. This value $p = p_c$ is taken as the critical density of cracks sufficient for fracture.

The same conclusion can be drawn in another way, by using the notion of a minimum 'live' (i.e. intact) cross-section undergoing a constant load with increase of defect concentration. With the growth of an average-sized cluster $S$ the length of undisturbed areas (pillars) naturally decreases and, in doing so, 'pillars' having a certain minimum size approach 0 at $p \to p_c$ (most likely as $1/S$). This tendency to a zero-value of the smallest intact volume (area) at $p \to p_c$ is, evidently, a precursor of a major fracture.

Actual disintegration of a body takes place when areas of disturbed continuity and not action radii overlap. In the author's opinion, this transition occurs momentarily, i.e. it is a fast breakdown of an already crushed medium by main fracture which in mechanics is defined as 'athermal' or 'forced' rupture. This phenomenon is not considered in the model, the latter being limited by the condition of reaching a critical prefracture state. Experimental results (section 5) support the idea that the percolation threshold for radius of action of cracks is reached simultaneously with the threshold of macrofracture. In the simplest model described, the role of the external load is reduced to the two following moments: (1) it must be sufficiently large to prevent 'healing' of interatomic bonds broken by thermal fluctuations, thus leading to the growth of $p$; (2) it causes disintegration of the system brought to the critical density of cracks, $p_c$. In section 4 the ways of building a more sophisticated model are presented.

It should be noted that, in principle, it is possible to pass from parameter $p$ to destruction kinetics (its time characteristic), but in doing so it is necessary to introduce complementary assumptions dealing essentially with the duration of a single act (Smythe and Wierman, 1978).

In isotropic percolation models, the interpenetration of an infinite cluster from one point (fracturing by propagation of a single crack) and as well as its formation by merging of isolated clusters (disperse fracture) requires the same critical concentration $p_c$.

## 4. Tension and shear

### *4.1. Role of system finiteness*

In the previous discussion the character of the effect was not defined absolutely. At the same time, it is clear that, while for sufficient tension to occur, infinite clusters and, consequently, the fracture surface can have a complex form (it must only be free from 'locks' preventing uncoupling), for shearing to occur this surface should be nearly flat (or nearly linear for $2D$).

The critical volume concentration of faults required for the formation of a 'film' IC (denoted as $p_{cf}^{3D}$) in a three-dimensional body exceeds the threshold value for conventional 'bulk' IC ($p_c^{3D}$) and according to Shklovsky and Efros (1979) it can be calculated for the $3D$-model of random sites by the equation

$$p_{cf}^{3D} \simeq p_c^{3D}\left[1 + \mathrm{d}(\bar{r}/h)^{1/\nu_3}\right] \quad (4)$$

where: $\bar{r}$ is the average distance between sites; d is a constant; and $\nu_3 = 0.9$, the critical index of a percolation radius. If $\bar{r} = h$, then

$$p_{cf}^{3D} \simeq p_c^{3D}(1 + \mathrm{d}) \quad (5)$$

Assuming that $p_{cf}^{3D}$ is always close to $p_c^{2D}$, i.e. the percolation threshold of two-dimensional systems, d can be shown to have a value in the order of unity, and the ratio $p_{cf}^{3D}/p_c^{3D} \simeq p_c^{2D}/p_c^{3D} \simeq 2$–$3$. If we assume that crack concentration is determined by stress, the ratio $p_{cf}^{3D}/p_c^{3D}$ corresponds to the ratio of rupture strength and shear strength, the latter also being 2–3, according to experimental results for rocks.

### *4.2. Anisotropically-correlated and cyclic models*

Presence of preferred directions along which percolation is enhanced (or hampered) noticeably influences the development of the percolation process. Such models can be named 'anisotropically-correlated'. The presence (or absence) of preferred directions particularly reveals itself in analysis of a network of cracks. Anisotropy of networks can be caused by non-hydrostaticity of load as well as by sample structure.

The simplest way of introduction of non-equivalence of directions into the percolation model is to pass from determined isotropic processes to probabilistic anisotropically-correlated models. For this purpose, a higher probability of defect formation in a certain direction is introduced, to be precise, in the direction perpendicular to that of least compression. One can try to attribute this probability to intensity coefficients.

In anisotropic models, the effect of system finiteness on sample properties is particularly marked. Although, in an infinite system, the main theory parameter (i.e. percolation threshold) does not change with the introduction of anisotropy (Shklovsky and Efros, 1979), in finite systems the threshold of percolation decreases appreciably in the direction of higher probability of crack formation. In numerical experiments using a computer the author has obtained for square lattices approximate estimates for the effect of degree of anisotropy of probabilities ($W$) on $p_c$ in two mutually-perpendicular directions corresponding to the maximal ($W_1$) and the minimal ($W_2$) compression at various $W_1/W_2$ values (Chelidze and Kolesnikov, 1982). It was found that at $W_1/W_2 < 10$ percolation threshold in the direction of maximal compression $p_c(W_1)$ does not change appreciably (by several percent), whence it follows that behaviour of characteristic functions and critical indices will also change only slightly. With further increase of $W_1/W_2$ an appreciable lowering of $p_c(W_1)$ is noted. At the limit, $W_1/W_2 \to \infty$, the first defect formed interpenetrates the whole system without branching with $p_c = L/l$ for $2D$. This is a minimum $p_c$ value which is reached only in the case when all disturbances are included in IC and the latter is rectilinear, the remainder of the lattice remaining intact. A more detailed description of the correlated model will be published at a later date. Here, I should like to note that results obtained in 'classical percolation theory' evidently retain their validity for slightly-correlated finite models.

Another interesting possibility for the application of percolation models of finite systems can be presented by models with 'healing'. It is known that fractures in rocks are often 'healed' either literally, through deposits from hydrothermal solutions, or effectively, due to increase of friction of edges with the increase of hydrostatic or decrease of porous pressure, etc. Consider the simplest case, a finite system in which equilibrium is established between newly-formed and healing cracks. Though the number of cracks in a body is constant, due to the random nature of places of their formation and healing, the configuration of a system of cracks will continuously change.

While in an infinite system the 'rupture' configuration would not be realized at $p < p_c$, for finite systems a certain Gaussian distribution of IC formation probability is characteristic at $p < p_c$ and, naturally, of its non-formation at $p > p_c$. Percolation probability $W(p_i)$ or relative frequency of formation of 'rupture' configuration $N(p_i)$ is the higher the less is system size $L$ and the greater is $p_i$. According to the theory of Gaussian random processes, the probability of 'breaking down' in the range $0 < p < p_i$ is

$$N(p_i) = W(p_i) = \Phi\left[(0 - \langle p_{cl}\rangle)/(D_p)^{1/2}\right] - \Phi\left[(p_i - \langle p_{cl}\rangle)/(D_p)^{1/2}\right] \quad (6)$$

where: $\Phi$ is the tabulated probability integral; $\langle p_{cl}\rangle$ is the mean value of percolation threshold at a given value of $L$; $D_p$ is the dispersion of percolation threshold $\langle p_{cl}\rangle$, which according to Levinstein et al. (1975) is dependent on $L$ and the dimensionality of the system

$$D_p = \mathrm{B}(\tilde{L} + \mathrm{C}_1)^{-1/\nu} \quad (7)$$

where: B and $C_1$ are constants; $\tilde{L} = L/l$, the reduced size of a system in lattice constant units; $\nu$ is the critical index of correlation radius, dependent on system dimensionality. $\tilde{L}$ and $p_i$ not being very small, eq. 7 can be simplified to

$$N(p_i) \simeq \Phi\left[(p_i - \langle p_{cl}\rangle)/(D_p)^{1/2}\right] \quad (8)$$

Assuming that time of realization of each configuration is constant and equals $\tau$, one can pass from a number of events to time and calculate average 'period' of formation of rupture configuration $\bar{T}$ (an analogue of duration of seismic cycle)

$$\bar{T}=\tau/N(p_i)=\tau/\Phi\left[(p_i-\langle p_{cl}\rangle)/B(\tilde{L}+C_1)^{-1/\nu}\right] \quad (9)$$

where correlation of frequency of ruptures (earthquakes), concentration of disturbances $p_i$, size $\tilde{L}$ and dimensionality $D$ of the system is of great interest.

## 5. Comparison of percolation model with experimental data

In this section are presented some experimental results attesting to the validity of the percolation model of fracture:

(1) Numerous data are available on acoustic emission (AE) according to which fracture of bodies without large biographic defects (in any case at those stages which precede the propagation of main fracture) is a multiple process, elementary acts of destruction being distributed diffusely throughout the bulk of the sample.

(2) The concept of critical prebreaking concentration of cracks (Scholz, 1968) found quantitative expression in the works of Zhurkov et al. (1977) where, on the basis of experimental results in acoustic emission and microscopy, an empirical concentration criterion of crack enlargement, $K \simeq N^{-1/3}/l$, is introduced, where $N$ is the concentration of elementary cracks (per $cm^3$), $N^{-1/3}$ is the average distance between elementary cracks and $l$ is crack length. In a percolation model of random sites, there is a completely analogous concentration criterion of IC formation, $N^{-1/3}/\Gamma_c \simeq C_2$. Here, $N^{-1/3}$ is the average distance between sites at a concentration ($N$) at which every site is involved in the sphere with radius $\Gamma_c$ circumscribed around any neighbouring site, $C_2$ is a constant. If we identify $l$ and $\Gamma_c$, both expressions become completely identical.

(3) The ratio of shear ($S_s$) and tension ($S_t$) strength of rocks approaches the ratio of concentrations $p_{cf}^{3D}/p_c^{3D}$, and can be explained by differences in IC topologies.

(4) We can compare the behaviour of precursors of fracture with that of precursors of an infinite cluster formation by functions of the type $S$, $L$ and $N_{cl}$.

Here, it is stressed that, in any comparison, one should take into consideration the specificity of the method of observation of a precursor. A precursor can be some function of the number of elementary cracks $p$ (or time $t$) as well as a derivative $(\mathrm{d}f(p)/\mathrm{d}p)$ or integral $(\int_{p_0}^{p} f(p)\,\mathrm{d}p)$; for example, notions 'AE activity' $(\mathrm{d}f(t)/\mathrm{d}t)$ and 'summary AE' $(\int_0^t f(t)\,\mathrm{d}t)$.

In order to compare theory with the experiment number of elementary events, $p$, must be recorded but this is very seldom done. Consider data on AE given by Zhurkov et al. (1977) who recorded separately the number of weak pulses which, in the author's opinion are identical to the number of elementary cracks or to parameter $p$ in percolation theory, as well as the number of strong pulses corresponding, in our mind, to a certain function $f(p)$ describing crack clusterizing. Reconstructing the plot of Zhurkov et al. (1977) in coordinates $f(p)$ versus $p$ and matching scales on axes $p$ and $f(p)$, we choose the best percolation function for $f(p)$. The principles of scaling are clear from

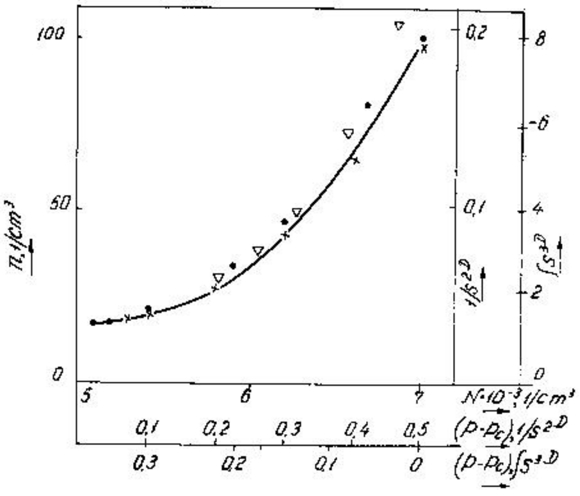


Fig. 2. Number of strong acoustic pulses $n$ in diabase versus weak pulses number $N$ (solid line) and percolation functions $1/S(p)^{2D}$ (crosses) and $\int S(p)^{3D}\,\mathrm{d}p$ (triangles). The scaling operation along the axis of ordinates is carried out at $(p-p_c)=0.3$ for $1/S^{2D}$-model and at $(p-p_c)=0.1$ for $\int S^{3D}\,\mathrm{d}p$ model. (Experimental data from Zhurkov et al., 1977.)

Fig. 2. It came out that the experimental curve can be approximated by function $1/S(p)^{2D}$ in the range $p_c < p < 1$ or by function $\int_0^p S(p)^{3D}\,dp$ in the range $0 < p < p_c$. The first interval corresponds to the assumption used in my first paper (Chelidze et al., 1979) that considerable AE begins only after IC formation. The second range ($0 < p < p_c$) corresponds to the model accepted in this paper in which the formation of macrofracture is associated with the condition when $p_c$ is reached.

Various percolation functions and precursors were also compared with experimental data from Mogi (1962), Scholz (1968) and others from the point of view of dependence of AE and dilatancy on relative pressure $P/P_{max}$ (Figs. 3 and 4). In these experiments, weak pulses (elementary cracks) were not recorded, therefore it was necessary to assume a linear relationship between $P/P_{max}$ and number of elementary events $p$.

Some known precursors of an earthquake were also analysed, for example, such as change of concentration of radon in underground waters before the earthquake in Tashkent in 1966 observed by Ulomov and Mavashev (1967) and statistical data on foreshock activity before strong earthquakes (Jones and Molnar, 1979)—see Figs. 4 and 5.

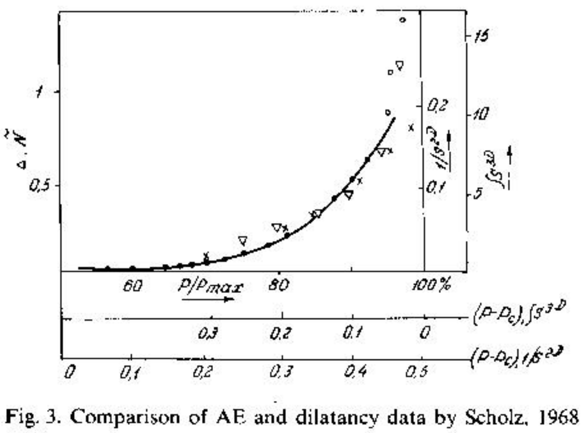


Fig. 3. Comparison of AE and dilatancy data by Scholz, 1968 (both are represented by points) with percolation model. Crosses show $1/S^{2D}$ and triangles $\int S^{3D}\,dp$-function. The scale along the vertical axis is chosen with regard to the point $P/P_{max} = 90\%$, where $(p - p_c)$ is assumed equal to 0.4 for $1/S^{2D}$ and 0.1 for $\int S^{3D}\,dp$-models. Solid curve—normal distribution function.

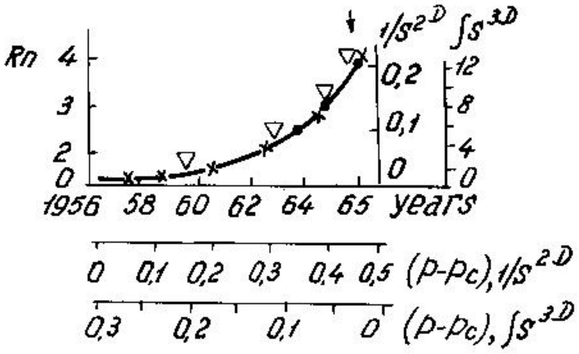


Fig. 4. Variation of radon content in underground water before Tashkent earthquake in 1966 (points) and percolation model predictions. Scaling is carried out at $(p-p_c)=0.5$ for $1/S^{2D}$-model and $(p-p_c)=0.1$ for $\int S^{3D}\,dp$-model.

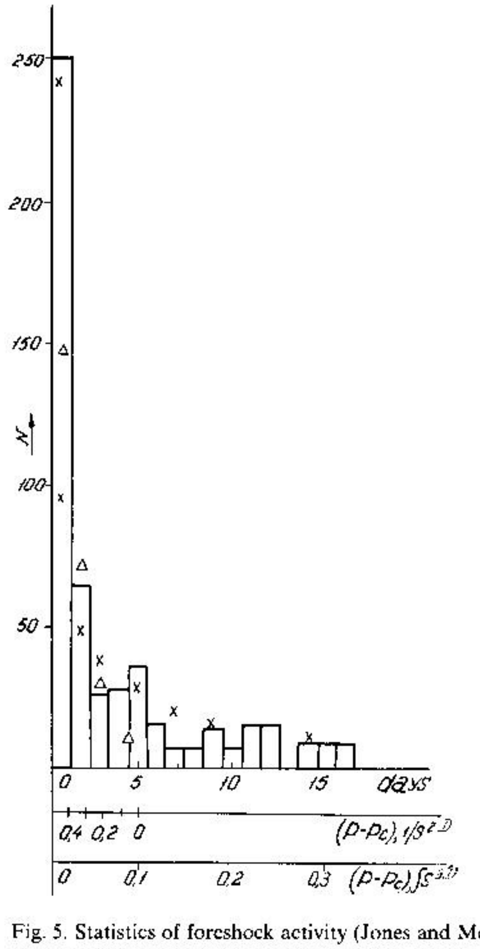


Fig. 5. Statistics of foreshock activity (Jones and Molnar, 1979) and percolation functions. Scaling is carried out at $(p-p_c)=0.5$ for $1/S^{2D}$-model and $(p-p_c)=0.2$ for $\int S^{3D}\,dp$-model.

The author has used as a condition the linear correlation of the crack concentration parameter $p$ with time. This assumption is valid for almost all types of material in the range of stationary or quasi-stationary creep (Rikitake, 1976) and is therefore justified. Radon concentration and foreshock activity can be satisfactorily approximated by functions $1/S(p)^{2D}$ and $\int_0^p S(p)^{3D}\,dp$ for corresponding ranges and resistance, by the function $1/G(p)^{2D} = 1/|p-p_c|^{1.0}$.

Thus, many precursors of fracture in laboratory experiments, as well as in the crust itself, can be described by definite percolation functions. The fact that AE, dilatancy and foreshock activity can be approximated by some function of quantity $S$ (mean size of clusters) obviously has certain physical meaning, i.e. that larger cracks (local clusters) are formed as the moment of formation of main fracture (IC) is approached.

(5) Of great interest is a situation arising when a three-dimensional system is fractured by shear. In this case formation of slip plane is preceded by formation of a system of interconnected cracks ('bulk' IC, see section 4). Consequently, before a shear fracture permeability for flow of gas, liquids, charges, etc. the so called 'transport' properties of the medium should increase sharply. Similar behaviour was observed in laboratory (Paterson, 1978).

## 6. Conclusions

Comparison of basic principles of the percolation theory with experimental results of physics of fracture leads to the conclusion that it is expedient to build a percolation model of fracture in two main versions, i.e. a model of interpenetration into infinity (Otsuka, 1972; Vere-Jones, 1976) and a model of multiple-crack formation in infinite and finite systems (see above).

In the author's opinion, new and interesting results can be obtained in this direction, namely:

(1) concentration criterion of multiple fracture is substantiated;

(2) an explanation is apparent for the difference between shear strength and rupture strength in terms of topology of an infinite cluster;

(3) the possibility of description of evolution of the process of crack formation by characteristic percolation functions is shown and consequently, the possibility of forecasting this process;

(4) characteristic features introduced by finiteness of a system are considered which result in the simplest of models of seismic cycles and anisotropically-correlated systems of fracture.

All this gives hope that the percolation model offers a new means for the quantitative description of fracture as a developing process, as well as for prediction of main fracture formation.

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